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Towards a New Height Datum for Uganda

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Abstract

A new height datum for Uganda is computed using the corrective surface principle. It is based on a combination of the Uganda Gravimetric Quasigeoid model (UGQ) 2014 and GNSS/levelling. UGQ2014 was derived from the Uganda Gravimetric Geoid model (UGG) 2014, which was computed from sparse terrestrial gravity data from the International Gravimetric Bureau, the 3 arc second Shuttle Radar Topography Mission digital elevation model and the GOCE – only global geopotential model GO_CONS_GCF_2_TIM_R5. The corrective surface was constructed based on 21 discrete GNSS/levelling points and then evaluated with 4 independent points. Three interpolation techniques were tested for the creation of the corrective surface with the Kriging method giving the lowest standard deviation and noise level suggesting that it is the best method for interpolation. In absolute terms, the Root Mean Square of the fit between the known and computed normal-orthometric heights based on the new height datum is 11cm, which is about 5cm (31%) better than using UGQ2014 alone. For relative heights an average precision of 29 ppm is computed for all baselines tested. Both the absolute and relative tests show that the new height datum satisfies the precision and accuracy requirements of third order precise levelling. Therefore, UGQ2014C represents a significant step towards the determination of a precise new height datum for Uganda.

1. Introduction

Traditionally the difference in height between two points A and B on the Earth's surface is practically measured using the technique of spirit levelling. However, it is well known that spirit levelling is a tedious and time consuming exercise especially in hilly terrain. For countries like Uganda where about 80% of the Fundamental Benchmarks (FBMs) were destroyed (Ssengendo, 2015), spirit levelling is additionally a very costly exercise. Nowadays the combined use of Global Navigation Satellite Systems (GNSS) and gravimetric quasigeoid models and/or geoid models can provide an alternative, which is easier and less costly than spirit levelling. The height combination problem is based on the simple geometrical relationship:

$$H^N = h - \zeta \qquad [1]$$

where H^N is the normal height, ζ is the height anomaly and h is the ellipsoidal height determined using GNSS. However, in practice the inherent simplicity of Equation (1) is never fulfilled due to random noise, datum inconsistencies and systematic distortions in the three height data sets. To minimize the effect of all the systematic biases the absolute differences can be fitted to parametric models through the use of least squares (Kiamehr & Sjöberg, 2005). The modelling options can range from simple plane-fit models to multiple regression equations of various orders. In general, the choice of the model depends on the distribution, quality and quantity of the GNSS/levelling data in the area of interest (Fotopoulos, 2013).

For the absolute case, the general form of the height combination problem can be represented by:

$$\Delta\zeta_i = a_i^T x + v_i \qquad [2]$$

where $\Delta\zeta_i$ is the difference between GNSS/levelling and model derived height anomalies, a_i is an mx1 vector of known coefficients depending on the parametric model used, x is a vector of the unknown parameters, v_i is a vector of residuals and m is the number of GNSS/levelling points.

In matrix notation, the least squares adjustment problem becomes

$$Ax = \Delta\zeta - v \qquad [3]$$

where A is the design matrix composed of one row a_i^T for each observation $\Delta\zeta_i$. The least squares solution to Equation (3) utilizing the mean of the squares of the residuals becomes

$$x = (A^T A)^{-1} A^T \Delta\zeta \qquad [4]$$

with residuals

$$\hat{v} = \Delta\zeta - A\hat{x} = [I - A(A^T A)^{-1} A^T]\Delta\zeta \qquad [5]$$

2. Determination of the Uganda Gravimetric Quasigeoid Model 2014

The determination of UGQ2014 by Ssengendo (2016) was based on Equation (6) below;

$$\zeta = N + (\zeta - N) \qquad [6]$$

where N is the geoid height extracted from UGG2014 (Sjöberg et al, 2015),$(\zeta - N)$ is the quasigeoid-geoid separation which was computed from Equations (7a, 7b & 7c) given by Sjöberg and Bagherbandi (2012)

$$\zeta - N = \frac{T(r_P,\Omega)}{\gamma_Q(\varphi)} - \frac{T^*\left(r_g,\Omega\right)}{\gamma_0(\varphi)} +$$

$$\boldsymbol{\zeta - N = \frac{T(r_P,\Omega)}{\gamma_Q(\varphi)} - \frac{T^*(r_g,\Omega)}{\gamma_0(\varphi)} + \frac{V^t_{bias}(r_P,\Omega)}{\bar{\gamma}(\Omega)}} \qquad [7a]$$

$$\text{with } \boldsymbol{T^*\left(r_g,\Omega\right) = \sum_{m=-n}^{n} T_{nm} Y_{nm}(\Omega)} \qquad [7b]$$

$$V^t_{bias}(r_P,\Omega) = 2\pi G \rho_0^t \sum_{n=0}^{n_{max}\Sigma nm(\Omega)} \sum_{m=-n}^{n} \left(H^2_{nm} + \frac{2}{3R} H^3_{nm}\right) \qquad [7c]$$

Here T is the disturbing potential at an arbitrary point (r, Ω), R is the Earth's mean radius, Y_{nm} are the fully normalized spherical harmonic functions of degree n and order m, T_{nm} are the fully normalized coefficients of the disturbing potential, n_{max} is the upper summation index of spherical harmonics, $\gamma_Q$is the normal gravity at the telluroid, γ_0 is the normal gravity at the reference ellipsoid, r_P is the geocentric radius of the surface point. $T^*(r_g, \Omega)$ in Eq. (7b) is the analytically continued external type harmonic series at the geoid where the true potential is not harmonic. The 3-D position is defined in the system of spherical coordinates(r, Ω), where r is the spherical radius and $\Omega = (\varphi, \lambda)$ is the spherical direction with the spherical latitude φ and longitude λ. $\frac{V_{bias}^t}{\bar{\gamma}}$ is the topographic bias which represents the error in the analytical downward continuation of the external gravitational potential inside the topographic masses (Sjöberg 2007) where ρ_0^t is the mean topographic mass density and the terms $\left\{\sum_{m=-n}^{n} H_{nm}^i Y_{nm}(\Omega): i = 1,2,3,\ldots\right\}$ define the spherical height functions $\left\{H_n^i: i = 1,2,3,\ldots.\right\}$; i.e.

$$H_n^i(\Omega) = \frac{2n+1}{4\pi} \iint_{\varphi} H^i(\Omega') \, P_n(t) d\Omega' = \sum_{m=-n}^{n} H_{nm}^i \, Y_{nm}(\Omega) \qquad [8]$$

where P_n is the Legendre polynomial of degree n with $t = cos\,\psi$ i.e. the cosine of the spherical distance between spherical directions Ω and Ω'.

3. New Height Datum based on a combination of UGQ2014 and GNSS/levelling

To determine the new height datum, the gravimetric quasigeoid model UGQ2014 was combined with GNSS/levelling data using the corrective surface idea. The role of the corrective surface is to provide an easier way of determining the normal height of a new point (*P*) such that:

$$H_P^N = h_P - \zeta_P^{UGQ} - (a_P^T \hat{x}) + v_P \qquad [9]$$

Here h_P is the ellipsoidal height from GNSS, ζ_P^{UGQ} is the height anomaly from the gravimetric quasigeoid model, the term in the brackets represents the effect of the corrective surface based on the latitude (φ_P) and longitude (λ_P) of point P and v_P is the least squares residual error. As discussed in Ssengendo (2015), the height system in Uganda is the normal-orthometric height system, which uses only the normal gravity field as an approximation of the Earth's gravity field to derive all the necessary gravity-related quantities hence avoiding the need for actual gravity observations along the levelling route. However, the zero reference surface for normal-orthometric heights is in reality not the quasigeoid but another poorly defined surface, which does not correspond to the quasigeoid meaning that normal heights are not coincident with normal-orthometric heights (Ssengendo, 2015). Nevertheless, the practical computation of the normal-orthometric heights is based on the assumption that the quasigeoid height is equivalent to the height anomaly as shown in Figure 1 below.

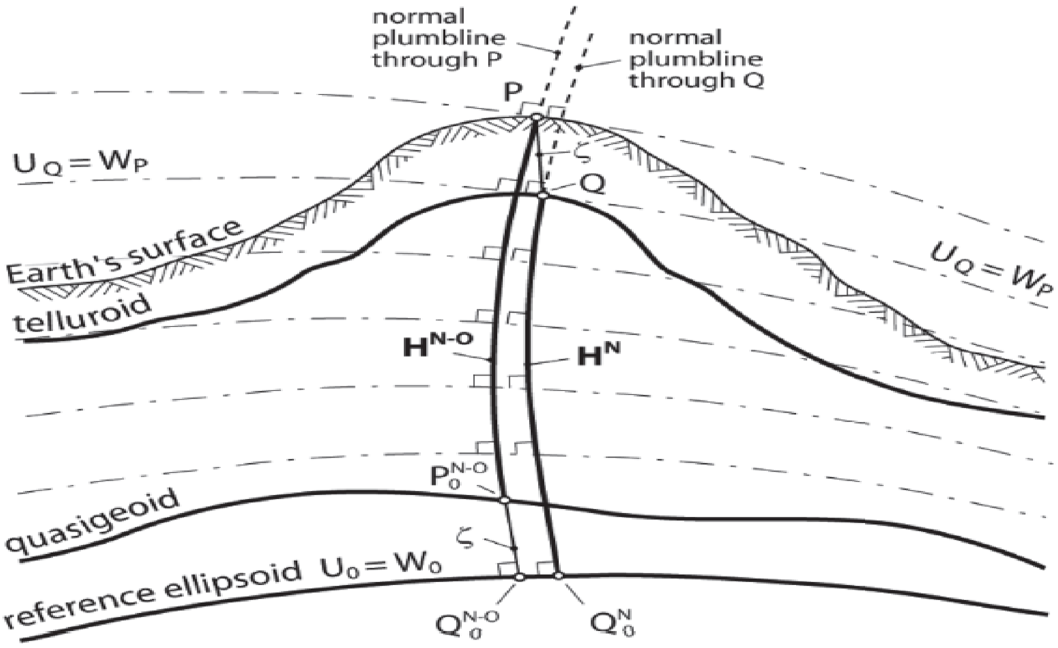

Figure 1: The normal height (**H**^{N}), normal-orthometric height **(H**$^{N-O}$**)** and their relationships to the height anomaly and quasigeoid height (Featherstone and Kuhn, 2006).

3.1. Selection of the interpolation method

In order to determine the corrective surface, the 12 GNSS/levelling points were sub-divided into 2 groups as shown in Figure 2. Group 1 with 8 GNSS/levelling points was used for the determination of the 4 parameters of the model, and group 2 with 4 points was used as an independent check of the precision of the corrective surface. Using $\hat{\boldsymbol{x}}$ and the latitudes and longitudes of the 21 discrete GNSS points (including the 13 points which were observed by GNSS but without normal-orthometric heights) the effect $a_P^T\hat{\boldsymbol{x}}$ for each point was computed. For the 8 GNSS/levelling points with the triplets of heights, the adjusted residual v_i was also computed using Equation (5).

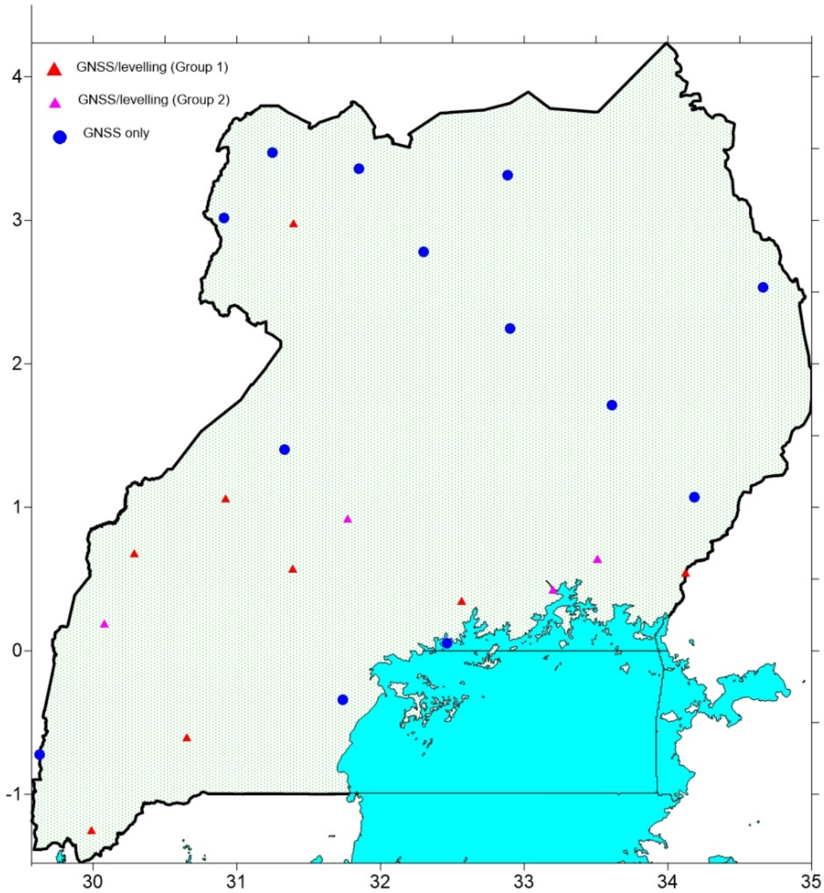

Figure 2: Network of GNSS/levelling and GNSS only points used for the generation of the corrective surface

Three methods of interpolation (Kriging, minimum curvature and radial basis function) were tested to determine the best method for the study. Briefly, Kriging is a geostatistitcal interpolation method that generates an estimated surface from a scattered set of points with z-values by assuming that the distance or direction between sample points reflects a spatial correlation that can be used to explain variation in the surface. The Kriging tool fits a mathematical function to a specified number of points, or all points within a specified radius, to determine the output value for each location. It involves exploratory statistical analysis of the data, variogram modeling, creation of the resultant surface and is most appropriate when the data is known to have a spatially correlated bias in distance or direction as often is the case in soil science and geology (ArcGIS for desktop, 2019). Minimum curvature is an inexact method of interpolation that is designed to ensure that the amount of surface curvature is as small as possible based on an iterative procedure that seeks to smooth the interpolated grid to a pre-specified parameter setting using a grid which is progressively made increasingly fine (Smith *et al*, 2018). Radial basis function is a deterministic interpolation method, which is similar to geostatistical interpolation methods like Kriging but without the benefit of prior analysis of variograms and the absence of any underlying assumptions regarding the behaviour of the input data points (Smith *et al,* 2018).

The selection of the best method for interpolation was based on which of the three methods had the lowest mean and standard deviation after interpolation. The minimum, maximum, mean and standard deviation of the grid points for each of the three methods are reported in Table 1 together with the statistics of the original $a_P^T\hat{\boldsymbol{x}}$ at the 21 GNSS/levelling points.

Table 1: Statistics of the Kriging, minimum curvature and radial basis function after interpolation (Unit: cm)

Method	Number of points	Min.	Max.	Mean	Std.
Original ($a_i^T\hat{\boldsymbol{x}}$)	21	-21.0	29.1	1.8	17.5
Kriging	141151	-21.0	37.2	4.0	18.5
Minimum Curvature	141151	-42.0	106.4	4.7	25.5
Radial Basis Function	141151	-21.2	40.1	4.4	19.3

From Table 1, it is clear that the minimum curvature method of interpolation is worse off than the other two methods. Kriging and radial basis function method produce results that are statistically not significantly different even though Kriging is slightly better. As highlighted above, the distribution of the points used is not ideal as it is skewed towards the southern part of the country. This potentially has a strong impact on the results from the radial basis function method hence making Kriging a slightly better method. In this study, the Kriging method was therefore selected for the interpolation procedures.

3.2. Determination of the corrective surface

To determine the corrective surface, the term $a_P^T\hat{x}$ was spatially modelled into a grid file using the Kriging interpolation method. The corrector surface presented in Figure 3 shows that there is a north-south trend with positive values observed in the north and negative values observed in south with minimum values of about ±5 cm in the central part of the country with average elevations of about 1100 m. The north-south trend observed is most likely a result of the number of GNSS/levelling points used as there were more points in the south than in the north, which was not ideal. The selection of the points was based on how many Fundamental Benchmarks (FBMs) of the Uganda Vertical Network (UVN) could be readily identified on the ground and with matching records (i.e. relevant height data). As discussed in Ssengendo (2015), only 14 FBMs out of the 70 that made up the UVN were identified on the ground in good condition although matching records could only be identified for only 11 FBMs. This uneven distribution therefore affected the resultant corrector surface.

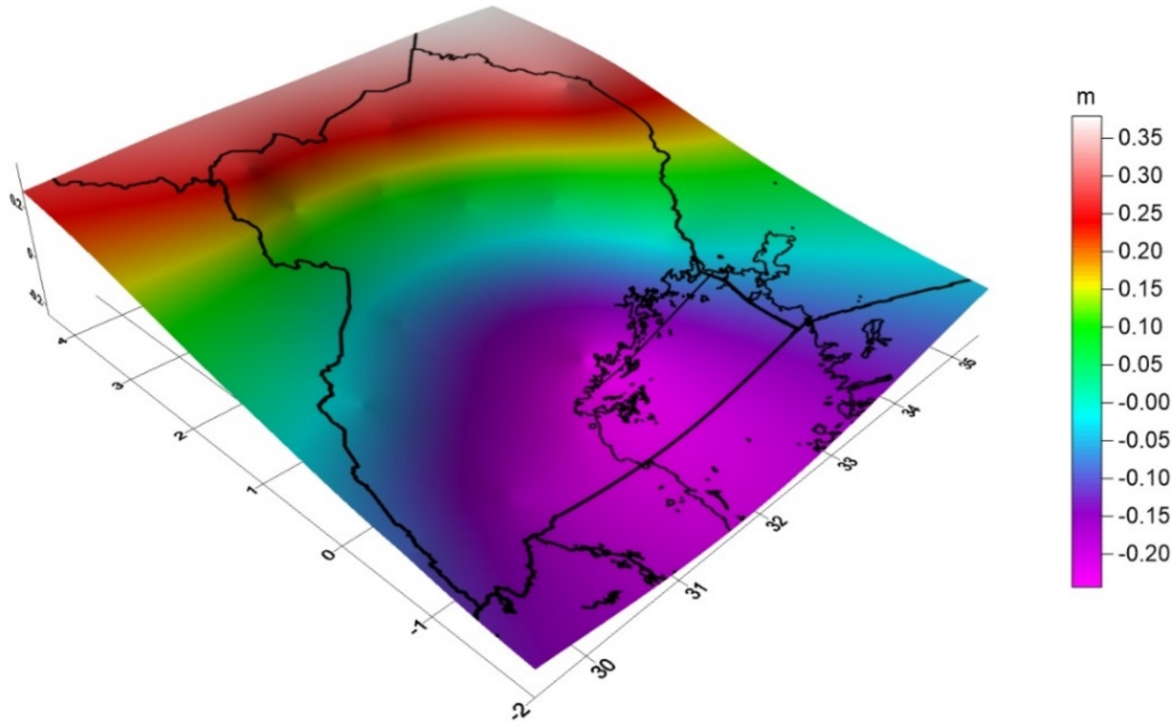

Figure 3: Corrector surface fit for Uganda based on the 4-parameter model and the Kriging interpolation method

4. New Height Datum

The new height datum is based on a re-arrangement of Equation (9) such that

$$H_P^{\text{N-O}} = h_P - \zeta^C \qquad [10]$$

where

$$\zeta^C = \zeta_P^{UGQ} + a_P^T \hat{x} + v_P \qquad [11]$$

is the 'corrected' height anomaly.

Equation (11) can be used to determine directly from GNSS the normal-orthometric height of any new point compatible with the vertical datum. By combining two grid files, that is, a grid of height anomalies from UGQ2014 and the corrective surface grid, a final grid of the new 'corrected' quasigeoid model (UGQ2014C) was obtained and is shown in Figure 4. A major question arises on how to scientifically interpret the 'corrected' quasigeoid model since it is no longer gravimetric in nature and therefore can no longer be used directly in geodetic, oceanographic and geophysical sciences (e.g. Vaníček and Christou, 1994). However, as noted by Featherstone (1998), the 'corrected' quasigeoid model provides a practical solution to the GNSS/levelling problem i.e. height determination directly from GNSS.

4.1. Evaluation of UGQ2014C

4.1.1. Absolute heights

As an external test of UGQ2014C, the normal-orthometric heights for 4 independent GNSS/levelling points, that is, Jinja FBM, Kasese FBM, Kiboga FBM and Nakalama FBM (see Figure 2) were computed and compared with the known normal-orthometric heights. The statistics are reported in Table 2 together with the statistics of using the gravimetric quasigeoid model (UGQ2014) only. By using the 'corrected' quasigeoid model, the RMS of the fit between the known and computed normal-orthometric heights is 11 cm, which is about 5 cm (31%) better than using the gravimetric quasigeoid model alone. Although all the 4 points used for the independent check are approximately on the same latitude, the performance of UGQ2014C highlights the significant contribution of the corrector surface as a practical solution to the GNSS/levelling problem.

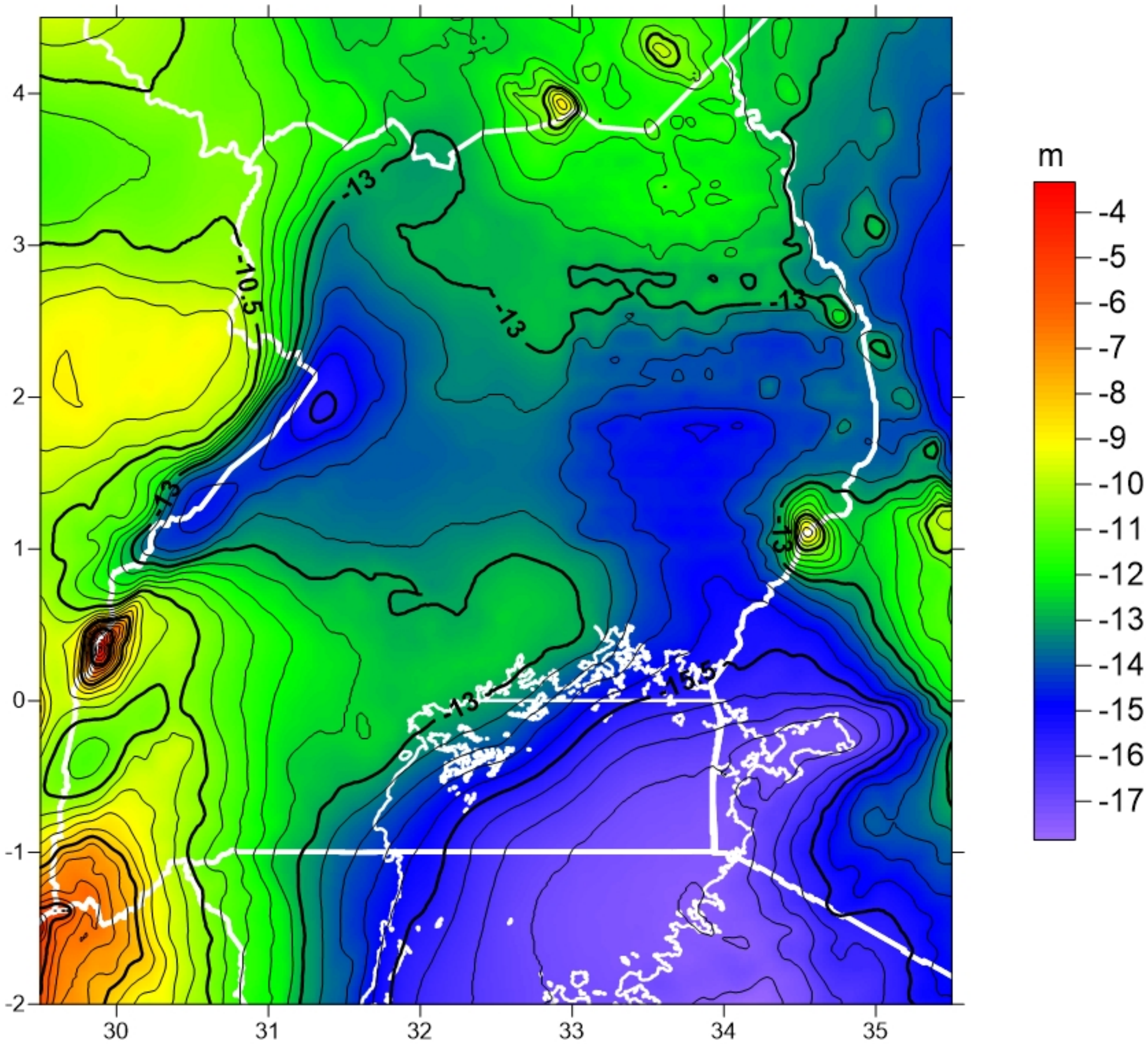

Figure 4: The new 'corrected' quasigeoid model over Uganda (contour interval =0.5)

Table 2: Validation of the 'corrected' quasigeoid model UGQ2014C and the gravimetric quasigeoid model UGQ2014 using 4 independent GNSS/levelling points (Unit: cm)

Model	Min.	Max.	Mean	Std.	RMS
UGQ2014	-5.2	30.5	9.0	15.4	16.1
UGQ2014C	-12.6	16.6	-1.6	12.6	11.1

4.1.2. Relative heights

Traditionally what is measured by spirit levelling is the height difference (ΔH) between any two points on the Earth's surface. Thus for selected baselines the normal-orthometric height difference $(\Delta H^{\text{N-O}})$ can also be computed from relative GNSS height differences (Δh) and 'corrected' height anomaly differences $\left(\Delta\zeta^{C}\right)$ as follows:

$$\Delta H^{\text{N-O}} = \Delta h - \Delta\zeta^{C} \qquad [12]$$

Then the height differences from Eq. (12) and spirit levelling can be compared so that the relative accuracy of UGQ2014C is computed in part per million (ppm) as follows:

$$ppm = \left|\frac{(\delta\Delta H^{\text{N-O}})}{k}\right| \qquad [13]$$

where k is the length of the baseline in km and $\delta\Delta H^{\text{N-O}}$ is the difference between the spirit levelled height difference $\left(\Delta H^{\text{N-O}}_{\text{LEV}}\right)$ and GNSS/UGQ2014C height difference $\left(\Delta H^{\text{N-O}}_{\text{GNSS/C}}\right)$ given by:

$$\delta\Delta H^{\text{N-O}} = \Delta H^{\text{N-O}}_{\text{GNSS/C}} \text{-} \Delta H^{\text{N-O}}_{\text{LEV}} \qquad [14]$$

The results of the relative test are presented in Table 3.

Table 3: Summary of the relative differences and ppm

Baseline		$k(km)$	$\Delta H^{\text{N-O}}_{\text{GNSS/C}}(m)$	$\Delta H^{\text{N-O}}_{\text{LEV}}(m)$	$\delta\Delta H^{\text{N-O}}(m)$	ppm
Kasese	Kiboga	200	-197.27	-197.27	0.00	0.03
Kiboga	Jinja	166	11.80	11.58	-0.002	13.14
Jinja	Nakalama	40	49.86	50.15	0.003	73.16
average						28.8

Overall an average precision of 29 ppm is computed for all the baselines. This meets the requirements of third order precise levelling. However, an important conclusion from the results is the presence of possible systematic biases in the levelling observations. This is evident in the *ppm* of the shortest baseline, which is much larger than the *ppm* of the longest baseline. Thus the 'corrected' quasigeoid model can also be used to provide a check of the known normal-orthometric heights of the benchmarks.

5. Conclusion

In spite of the few GNSS/levelling points used in the generation of the new height datum, it is encouraging that both the absolute and relative tests show that UGQ2014C satisfies the precision and accuracy requirements of third order precise levelling. Therefore, UGQ2014C represents a significant step towards the determination of a precise new height datum, which can be used as a practical solution for the determination of normal-orthometric heights directly from GNSS. With a denser and homogeneous network of GNSS/levelling points it is possible that the achievable accuracy from UGQ2014C can be improved to reach the second class order of precise levelling.

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